Name ____ Test 1, Fall 2020

1) Multiply the two matrices below or state why they cannot be multiplied. (15 points)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1+6+15 & 2+18 \\ 12+5 & 6 \\ 2+6+5 & 4+6 \end{bmatrix} = \begin{bmatrix} 22 & 20 \\ 17 & 6 \\ 13 & 10 \end{bmatrix}$$

2) Find the null space of the matrix below. (16 points)

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

$x_1 + 2x_2 = 0$
$x_1 = -2x_2$

$x_3 - 6x_4 = 6$
$x_3 = 6x_4$

$$\left\{ \begin{bmatrix} -2x_2 \\ x_2 \\ 6x_4 \\ x_4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \end{bmatrix} x_4 : x_2, x_4 \in \mathbb{R} \right\} = span\left(\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \end{bmatrix} \right\} \right)$$

3) Reduce the matrix below to reduced row echelon form. (16 points)

$$\begin{bmatrix} 4 & 8 & 4 & 8 \\ 0 & 2 & 4 & 6 \\ 1 & 2 & 1 & 0 \\ 1 & 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 & 4 & 8 \\ 0 & 2 & 4 & 6 \\ 1 & 2 & 1 & 0 \\ 1 & 4 & 5 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & 4 & 6 \\ 1 & 2 & 1 & 0 \\ 1 & 4 & 5 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & -2 \\ 1 & 4 & 5 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & -2 \\ 0 & 2 & 4 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 2 & 4 & 4 \end{bmatrix}$$

$R_1 \to \frac{1}{4}R_1$ $R_3 \to R_3 - R_1$ $R_4 \to R_4 - R_1$ $R_2 \to \frac{1}{2}R_2$

$$\sim_R \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 2 & 4 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$ $R_4 \to R_4 - 2R_2$ $R_3 \to -\frac{1}{2}R_3$ $R_1 \to R_1 + 4R_3$

$R_2 \to R_2 - R_3$

$R_4 \to R_4 + 2R_2$

4) Answer the questions below (3 points each)

A) Let A be a 6×6 matrix that is a product of elementary matrices. How many solutions does the equation $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 2 \end{bmatrix}$ have?

1

B) Suppose A is a 4×7 matrix whose column space has dimension 3. When A is row reduced, how many rows of zeroes does it have?

1

C) Suppose A is a 6×4 matrix. When row reduced, it has 2 pivots. How many solutions does the equation $A\vec{x} = \vec{0}$ have?

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D) Suppose A is a 6×4 matrix. When row reduced, it has 4 pivots. How many solutions does the equation $A\vec{x} = \vec{0}$ have?

1

E) Let A be a 5×5 matrix. Assume $A\vec{x} = \vec{0}$ has infinitely many solutions, but $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ has no solutions. What is the maximum number of pivots of A, after it is row reduced?

4

5) For each of the following, answer whether or not the two matrices can be multiplied. Answer “Y” for yes and “N” for no. (8 points)

Y or N $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

No

Y or N $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Yes

Y or N $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Yes

Y or N $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Yes

6) Solve the matrix equation below for X. Assume all matrices are of compatiable sizes and invertible.
(8 points)

$$AX + BX = C$$

$AX + BX = C$
$(A + B)X = C$
$X = (A + B)^{-1}C$

The following row reduction may be helpful for the problems on this page.

$$\begin{bmatrix} 3 & 2 & 1 & 4 & 5 \\ 1 & 1 & 0 & 1 & 1 \\ 6 & 4 & 2 & 12 & 10 \\ 1 & 1 & 0 & 0 & 0 \\ 8 & 6 & 2 & 14 & 12 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

7) Find the column space of the matrix below, avoid redundant vectors when possible. (7 points)

$$\begin{bmatrix} 3 & 2 & 1 & 4 & 5 \\ 1 & 1 & 0 & 1 & 1 \\ 6 & 4 & 2 & 12 & 10 \\ 1 & 1 & 0 & 0 & 0 \\ 8 & 6 & 2 & 14 & 12 \end{bmatrix}$$

$$span\left(\left\{\begin{bmatrix} 3 \\ 1 \\ 6 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 12 \\ 0 \\ 14 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 10 \\ 0 \\ 12 \end{bmatrix}\right\}\right)$$

8) What is the dimesion of the vector space below?. (7 points)

$$span\left(\left\{\begin{bmatrix} 3 \\ 1 \\ 6 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 12 \\ 0 \\ 14 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 10 \\ 0 \\ 12 \end{bmatrix}\right\}\right)$$

4

9) Find the inverse of the matrix below. (8 points)

$$\begin{bmatrix} 0.5 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\left[\begin{matrix} 0.5 & 1 & 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{matrix}\right] \sim_R \left[\begin{matrix} 1 & 2 & 0 & 2 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{matrix}\right] \sim_R \left[\begin{matrix} 1 & 2 & 0 & 2 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{matrix}\right]$$

$$\sim_R \left[\begin{matrix} 1 & 0 & -2 & 6 & -2 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{matrix}\right] \sim_R \left[\begin{matrix} 1 & 0 & -2 & 6 & -2 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{matrix}\right] \sim_R \left[\begin{matrix} 1 & 0 & -2 & 6 & -2 & 0 \\ 0 & 1 & 0 & -4 & 2 & -1 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{matrix}\right]$$

$$\sim_R \left[\begin{matrix} 1 & 0 & 0 & 10 & -4 & 2 \\ 0 & 1 & 0 & -4 & 2 & -1 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{matrix}\right]$$

$$\begin{bmatrix} 0.5 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 10 & -4 & 2 \\ -4 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$